Int. J. Heat Mass Transfer. Vol. 36, No. 14, pp. 3547–3554, 1993
Printed in Great Britain

0017–9310/93 \$6.00 + 0.00

Mixed convection from an isothermal horizontal plate moving in parallel or reversely to a free stream

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(*Received* 20 *August* 1992 *and in final form* 25 *January* 1993)

Abstract—This paper develops universal formulation for a general mixed convection problem in which the solid surface of an isothermal horizontal plate and the ambient fluid are both in motion. Similarity and nonsimilarity equations of six special convection systems can be reduced readily from the universal formulation. Very accurate numerical solutions and comprehensive correlations for $0.01 \leqslant Pr \leqslant 10\,000$ are presented over the entire domain of mixed convection and for any relative velocity between the plate and the free stream. The effects of buoyancy and relative velocity on the flow field, temperature field, surface friction, and heat transfer rate are clearly illustrated for a plate moving in parallel or reversely to the free stream for the buoyancy assisting and opposing cases.

1. INTRODUCTION

MIXED CONVECTION of a free stream flowing over a stationary, horizontal plate has been studied firstly by Mori [1], and by Sparrow and Minkowycz [2] in the early years of the 1960s. Since then very extensive works on this problem have been reported. A literature review was made by Schneider and Wasel [3]. Recently, Lin and his coworkers [4] compared the scope of mixed convection intensity and Prandtl number as well as the solution methods of the previous investigations. They extended the method of Raju *et al.* [5], which gave results over the entire range of mixed convection for $0.1 \leqslant Pr \leqslant 10$, to fluids of any Prandtl number between 0.001 and 10 000. They also presented comprehensive correlations [6] for predicting heat transfer and wall friction.

Another type of mixed convection, i.e. mixed convection over a moving horizontal plate in a quiescent ambient fluid, has also been investigated [7–9]. Recently, Karwe and Jaluria [9] have reviewed thoroughly the literatures on thermal transport from a moving plate.

In many manufacturing processes of film and plate, such as rolling, extrusion and drawing, the flow and thermal fields are strongly affected by the external flow, the movement of the solid surface, and the buoyancy arising from temperature difference. These practical processes can be modeled as a general mixed convection problem: a combined system of the natural convection and the forced convection due to the simultaneous movement of the solid surface and the ambient fluid. The general mixed convection model includes six subsystems: (1) natural convection over a horizontal plate; (2) forced convection on a stationary plate; (3) forced convection on a moving plate; (4) forced convection of a moving plate in a free stream [10–13]; (5) mixed convection over a stationary plate; and (6) mixed convection over a moving plate.

The present work studies the general mixed convection on an isothermal horizontal plate which moves in parallel or reversely to a free stream. We analyze the problem by introducing appropriate mixed convection parameter, relative velocity parameter, and other transformation variables. A universal formulation of the general mixed convection is then derived, from which the system equations of the afore-mentioned six special cases can be obtained readily.

2. ANALYSIS

Consider the mixed convection boundary-layer flow over a horizontal flat plate which moves continuously from a slot at a constant velocity u_s ($u_s \geqslant 0$). The plate moves either in parallel or reversely to a free stream of uniform velocity u_∞ ($u_\infty \geqslant 0$). Figure 1 shows a diagram of this system. The mixed convection boundary layer flow arises due to the interaction of the free stream, the movement of the plate, and the streamwise pressure gradient caused by the buoyancy force from the temperature difference between the surface temperature T_s and the ambient fluid temperature T_∞. Utilizing the usual Boussinesq assumption, the governing laminar boundary layer equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

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NOMENCLATURE

C_f	local friction coefficient, $2\tau_s/\rho u_\infty^2$	ζ	mixed convection parameter, λ_N/λ_F
f	dimensionless stream function, $\psi/\alpha\lambda$	η	dimensionless coordinate, $(y/x)\lambda$
g	gravitational acceleration	θ	dimensionless temperature, $(T-T_\infty)/(T_s-T_\infty)$
h	local heat transfer coefficient	λ	$\lambda_F+\lambda_N$
k	thermal conductivity of fluid	λ_F	$(\sigma\ Re_s+\omega\ Re_\infty)^{1/2}$
m, n	constant exponent	λ_N	$(\phi\ Ra)^{1/5}$
Nu	local Nusselt number, hx/k	ν	kinematic viscosity
p	pressure	ξ	mixed convection parameter, $\zeta/(1+\zeta)$
Pr	Prandtl number, ν/α	π	dimensionless pressure, $px^2/(\rho\alpha^2\lambda^4)$
Ra	Rayleigh number, $g\beta(T_s-T_\infty)x^3/\alpha\nu$	ρ	density
Re_s	Reynolds number, $u_s x/\nu$	σ	$Pr^2/(1+Pr)$
Re_∞	Reynolds number, $u_\infty x/\nu$	τ_s	surface shear stress
T	fluid temperature	ϕ	$Pr/(1+Pr)$
u, v	velocity components in x and y direction	ψ	stream function
x, y	horizontal and vertical coordinates.	ω	$Pr/(1+Pr)^{1/3}$.

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ	parameter of relative velocity, $[1+(\omega u_\infty)/(\sigma u_s)]^{-1}$

Subscripts

F	forced convection
N	natural convection
s	at the plate surface
∞	far from the plate surface.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y}\pm g\beta(T-T_\infty) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \quad (4)$$

where the plus sign on the last term of equation (3) represents the case of buoyancy assisting flow above a heated horizontal plate due to a favorable pressure gradient; and the minus sign applies to the buoyancy opposing flow above a cooled plate. The boundary conditions can be stated as

$$u = \pm u_s,\quad v = 0,\quad T = T_s \quad \text{at } y = 0 \quad (5)$$

$$u = u_\infty,\quad T = T_\infty,\quad p = 0 \quad \text{as } y \to \infty. \quad (6)$$

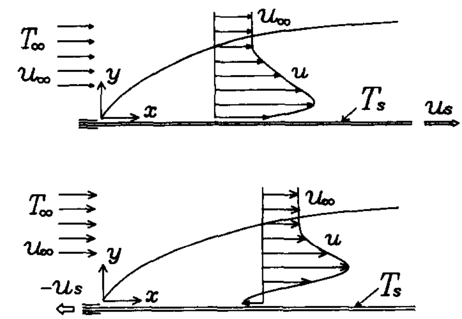

FIG. 1. Schematic diagram of mixed convection flow over parallel and reverse moving plates in a free stream.

The boundary condition of $u = +u_s$ pertains to the case of a plate moving in parallel to the free stream, while $u = -u_s$ represents the case of a reverse moving plate.

To analyze the convection problem in which both the plate and the ambient fluid are in motion, we define a parameter of relative velocity between the moving plate and the free stream as

$$\gamma = [1+(\omega u_\infty)/(\sigma u_s)]^{-1} \quad (7a)$$

$$= [1+(\omega Re_\infty)/(\sigma Re_s)]^{-1} \quad (7b)$$

where σ and ω are functions of Prandtl number:

$$\sigma = Pr^2/(1+Pr) \quad (8)$$

$$\omega = Pr/(1+Pr)^{1/3} \quad (9)$$

and

$$Re_s = u_s x/\nu,\quad Re_\infty = u_\infty x/\nu \quad (10)$$

are the Reynolds numbers based on the surface velocity and the free stream velocity, respectively.

The mixed convection parameter that measures the relative importance of natural convection and forced convection is proposed as

$$\zeta = \lambda_N/\lambda_F \quad (11)$$

where

$$\lambda_F = (\sigma Re_s+\omega Re_\infty)^{1/2} \quad (12)$$

and

$$\lambda_N = (\phi Ra)^{1/5} \quad (13)$$

with

$$\phi = Pr/(1+Pr) \tag{14}$$

and the Rayleigh number

$$Ra = g\beta(T_s - T_\infty)x^3/\alpha\nu. \tag{15}$$

To facilitate the numerical solution, another mixed convection parameter is defined as

$$\xi = (1+\lambda_F/\lambda_N)^{-1} = \zeta/(1+\zeta). \tag{16}$$

The entire mixed convection domain is then converted from $0 \leqslant \zeta \leqslant \infty$ to $0 \leqslant \xi \leqslant 1$.

In addition, a dimensionless coordinate is defined as

$$\eta = (y/x)\lambda \tag{17}$$

where

$$\lambda = \lambda_F + \lambda_N \tag{18a}$$

$$= \lambda_F/(1-\xi) \tag{18b}$$

$$= (\sigma Re_s)^{1/2}/[\gamma^{1/2}(1-\xi)] \tag{18c}$$

$$= (\omega Re_\infty)^{1/2}/[(1-\gamma)^{1/2}(1-\xi)] \tag{18d}$$

$$= (\phi Ra)^{1/5}/\xi. \tag{18e}$$

We also define a dimensionless stream function

$$f(\xi,\eta) = \psi/(\alpha\lambda), \tag{19}$$

a dimensionless temperature

$$\theta(\xi,\eta) = (T-T_\infty)/(T_s-T_\infty), \tag{20}$$

and a dimensionless pressure

$$\pi(\xi,\eta) = px^2/(\rho\alpha^2\lambda^4). \tag{21}$$

By using the newly defined dimensionless parameters and variables of equations (7)–(21), the governing equations (1)–(4) can be transformed into

$$Pr\, f''' + \frac{5+\xi}{10} ff'' - \frac{\xi}{5} f'f' + \frac{5-\xi}{10}\eta\pi' - \tfrac{2}{5}\xi\pi$$
$$= \tfrac{1}{10}\xi(1-\xi)\left[f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi} + \frac{\partial \pi}{\partial \xi}\right] \tag{22}$$

$$\pi' = \pm(1+Pr)\xi^5\theta \tag{23}$$

$$\theta'' + \frac{5+\xi}{10} f\theta' = \tfrac{1}{10}\xi(1-\xi)\left[f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right]. \tag{24}$$

The transformed boundary conditions are

$$f(\xi,0) = 0, \quad f'(\xi,0) = \pm\gamma(1-\xi)^2/\phi, \quad \theta(\xi,0) = 1 \tag{25}$$

$$f'(\xi,\infty) = (1-\gamma)(1-\xi)^2(1+Pr)^{1/3},$$
$$\theta(\xi,\infty) = 0, \quad \pi(\xi,\infty) = 0. \tag{26}$$

In the above equations, primes denote partial differentiations with respect to η.

Equations (22)–(26) are the universal formulation for laminar mixed convection over an isothermal horizontal plate. Equations of the following six convection subsystems are readily reducible from the universal equations by setting proper values of the parameters ξ and γ: (1) natural convection on a horizontal plate ($\xi = 1$); (2) forced convection of a stationary plate in a free stream ($\xi = 0$, $\gamma = 0$); (3) forced convection of a moving plate in a quiescent ambient fluid ($\xi = 0$, $\gamma = 1$); (4) forced convection of a moving plate in a free stream ($\xi = 0$, $0 \leqslant \gamma \leqslant 1$); (5) mixed convection of a stationary plate in a free stream ($\gamma = 0$, $0 \leqslant \xi \leqslant 1$); (6) mixed convection of a moving plate in a quiescent ambient fluid ($\gamma = 1$, $0 \leqslant \xi \leqslant 1$).

3. NUMERICAL METHOD

The general mixed convection equations (22)–(24) subject to the boundary conditions (25) and (26) were solved numerically by the well-known Keller's Box method. The implicit Box scheme is described in detail in ref. [14]. The numerical integration started at $\xi = 0$ and marched step-by-step with $\Delta\xi = 0.01$ to $\xi = 1$. The step size of η-coordinate, $\Delta\eta$, and the edge of the boundary-layer, η_∞, are adjusted for different range of Pr. A similar numerical procedure was described in ref. [15]. The accuracy of the numerical solutions has been verified by comparing with the results [3–5, 13] of the six special convection systems mentioned above. No analytical or experimental data are available for comparison since the studied system, not including the special cases, has never been investigated previously. However, all the factors that might introduce error are eliminated in the formulation and numerical procedures, except the laminar boundary layer approximation.

4. RESULTS AND DISCUSSION

4.1. *Velocity profiles*

The velocity component u is related to $f'(\xi, \eta)$ by

$$u = (\alpha/x)\lambda^2 f'(\xi,\eta). \tag{27}$$

Therefore, the function $f'(\xi, \eta)$ represents a dimensionless velocity $u/(\alpha/x)\lambda^2$. For the buoyancy assisting flow on a parallel moving plate, Fig. 2 shows the step-by-step variations of the profiles of the dimensionless velocity $f'(\xi, \eta)$ from the limiting case of a stationary plate ($\gamma = 0$) to the other limiting case of a moving plate in a quiescent ambient fluid ($\gamma = 1$). For the special case of $u_s = u_\infty$ ($\gamma = 0.3295$ for $Pr = 0.7$), the velocity profiles are uniform at the forced convection dominant region, $\xi < 0.3$, as can be seen from Fig. 2(b). Figure 2 also shows the gradual conversion of the velocity profiles from the forced convection limit to the natural convection limit as ξ varies from 0 to 1.

For the buoyancy opposing flow on a parallel moving plate, the inertia flow is retarded by the adverse pressure gradient arising from the buoyancy force. Figure 3 shows that the dimensionless velocity $f'(\xi, \eta)$ decreases slightly as buoyancy parameter ξ increases from 0 to 0.4, as expected. Convergent numerical solu-

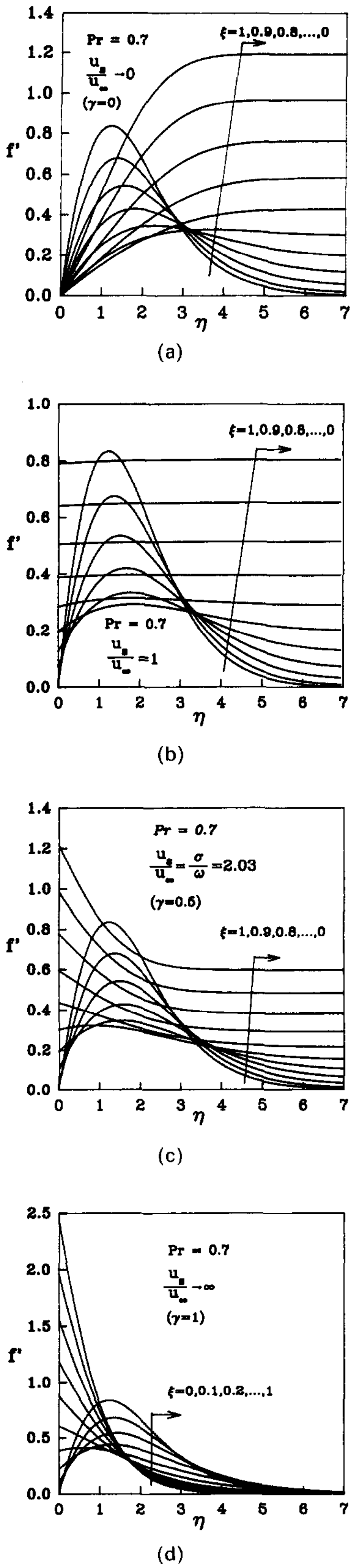

FIG. 2. Variations of the dimensionless velocity profiles for the buoyancy assisting flow on a parallel moving plate, $Pr = 0.7$, (a) $u_s/u_\infty \to 0$ ($\gamma = 0$); (b) $u_s/u_\infty = 1$ ($\gamma = 0.3295$); (c) $u_s/u_\infty = \omega/\sigma = 2.03$ ($\gamma = 0.5$); (d) $u_s/u_\infty \to \infty$ ($\gamma = 1$).

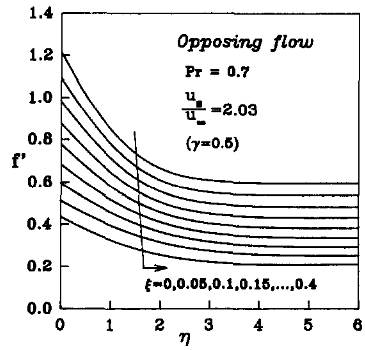

FIG. 3. Profiles of the dimensionless velocity for the buoyancy opposing flow on a parallel moving plate.

tions cannot be obtained for higher buoyancy force, for example $\xi > 0.4$ for $Pr = 0.7$ and $\gamma = 0.5$. Physically, this is due to the breakdown of boundary layer approximation [3] when the adverse pressure gradient is larger than a certain critical value. In that case, the system should be modeled by the whole conservation equations instead of the approximate boundary layer ones.

4.2. *Temperature profiles*

For the case of a plate moving in parallel to the free stream, representative dimensionless temperature profiles for $Pr = 0.7$ and $u_s/u_\infty = \sigma/\omega = 2.03$ ($\gamma = 0.5$) are shown in Fig. 4 for the buoyancy assisting and

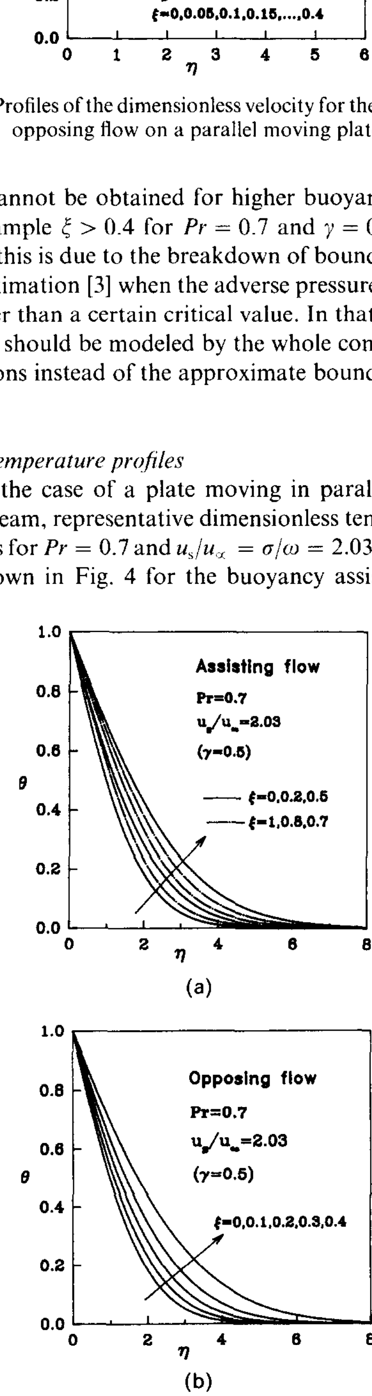

FIG. 4. Representative dimensionless temperature profiles for (a) buoyancy assisting flow; (b) buoyancy opposing flow on a parallel moving plate.

opposing flows. The temperature profiles for other values of Prandtl number and relative velocity parameter are similar to that shown in this figure, and thus are omitted here to conserve space. Figure 4 shows that, for the case of buoyancy assisting flow, the fluid temperature increases as the buoyancy parameter ξ increases from 0 to 0.5 but decreases as ξ increases from 0.6 to 1. For the buoyancy opposing flow, the fluid temperature increases as ξ increases from 0 to 0.4.

For the case of a reverse moving plate at low speed ($\gamma = 0.04$), the temperature profiles are similar to that in Fig. 4 for the parallel moving plate. For the buoyancy opposing flow over a reverse moving plate, the convergent solutions can only be obtained in the region of $0 \leqslant \xi \leqslant 0.3$ due to the unfavorable pressure gradient and the induced reverse flow by the moving plate.

4.3. *Surface friction*

The local friction coefficient

$$C_f = 2\tau_s/(\rho u_\infty^2) \tag{28}$$

is related to the numerical results of $f''(\xi, 0)$ by

$$C_f Re_x^{1/2} = 2\phi^{1/2}(1-\gamma)^{-3/2}(1-\xi)^{-3}f''(\xi,0). \tag{29}$$

For the case of a flat plate moving in parallel to a free stream, the variations of $C_f Re_x^{1/2}$ with the mixed convection parameter ζ for some specified relative velocity parameter γ are shown in Fig. 5 for the buoyancy assisting and opposing flows. Figure 5 shows that, for

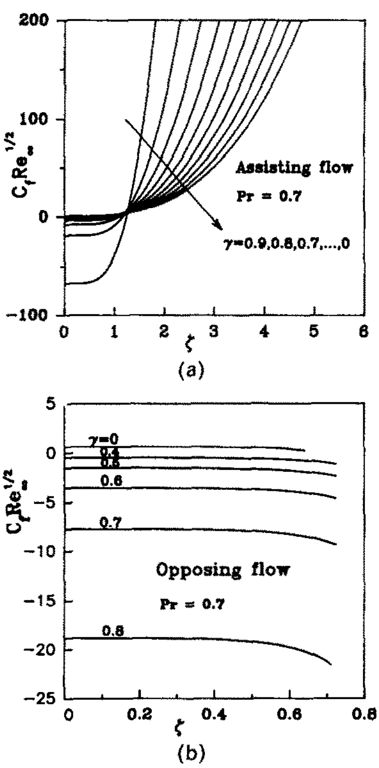

FIG. 5. Variations of $C_f Re_x^{1/2}$ with ζ for (a) buoyancy assisting flow; (b) buoyancy opposing flow on a parallel moving plate.

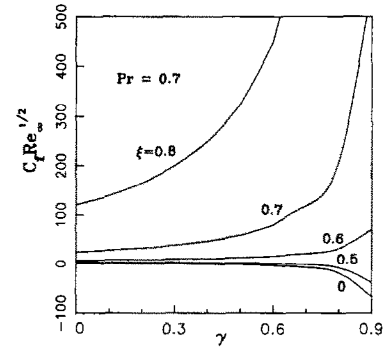

FIG. 6. Variations of $C_f Re_x^{1/2}$ with γ for buoyancy assisting flow on a parallel moving plate, $Pr = 0.7$.

any relative velocity, $C_f Re_x^{1/2}$ increases as ζ increases for assisting flow. On the contrary, $C_f Re_x^{1/2}$ decreases with increasing ζ for opposing flow. Figure 5 also reveals that, at the region where natural convection is dominant, $C_f Re_x^{1/2}$ increases with increasing γ. However, at the forced convection dominant region ($\zeta < 1.2$ or $\xi < 0.55$), $C_f Re_x^{1/2}$ decreases from a positive value (0.33206) to 0 as γ increases from 0 to that equivalent to $u_s = u_\infty$ (e.g. $\gamma = 0.3295$ for $Pr = 0.7$). Further increase of γ causes negative velocity gradient at the fluid–solid interface. The different tendency of the variations of $C_f Re_x^{1/2}$ with γ for $\xi > 0.55$ and $\xi < 0.55$ is also shown clearly in Fig. 6.

For the case of a plate moving reversely to the free stream, Fig. 7 shows that $C_f Re_x^{1/2}$ increases with increasing ζ for the buoyancy assisting flow but decreases with increasing ζ for the buoyancy opposing flow.

4.4. *Heat transfer rate*

The heat transfer rate from an isothermal plate to the ambient fluid can be calculated from the local Nusselt number, $Nu = hx/k$, which is related to the numerical results of $\theta'(\xi, 0)$ by

$$Nu = -\lambda\theta'(\xi,0). \tag{30}$$

Equation (30) can be rewritten as

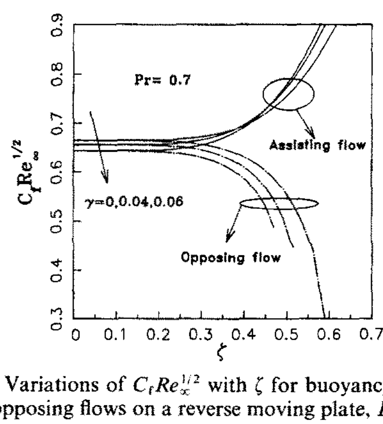

FIG. 7. Variations of $C_f Re_x^{1/2}$ with ζ for buoyancy assisting and opposing flows on a reverse moving plate, $Pr = 0.7$.

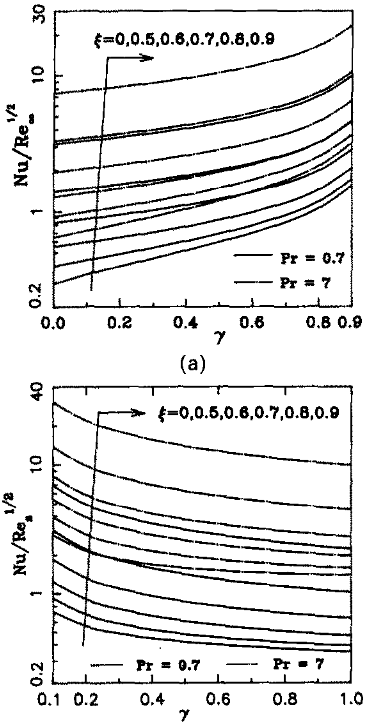

FIG. 8. Effects of relative velocity and buoyancy on heat transfer rate for buoyancy assisting flow on a parallel moving plate: (a) $Nu/Re_{\infty}^{1/2}$ vs γ; (b) $Nu/Re_s^{1/2}$ vs γ.

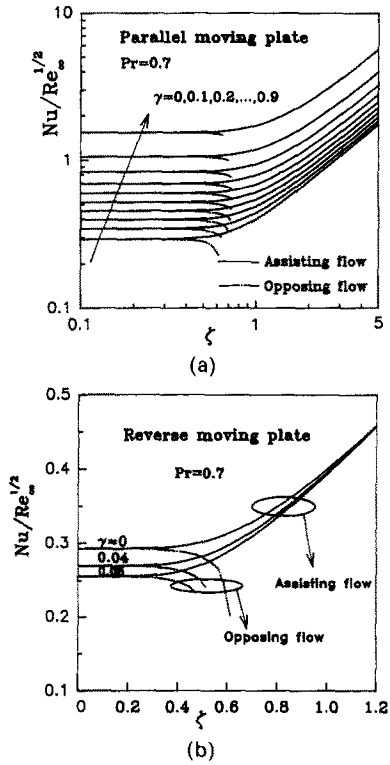

FIG. 10. Variations of $Nu/Re_{\infty}^{1/2}$ with ζ: (a) parallel moving plate; (b) reverse moving plate.

$$Nu/Re_{\infty}^{1/2} = \omega^{1/2}(1-\gamma)^{-1/2}(1-\xi)^{-1}[-\theta'(\xi,0)] \tag{31a}$$

or

$$Nu/Re_s^{1/2} = \sigma^{1/2}\gamma^{-1/2}(1-\xi)^{-1}[-\theta'(\xi,0)]. \tag{31b}$$

For the buoyancy assisting flow over a plate moving in parallel to the free stream, the variations of Nusselt number with the relative velocity parameter γ is presented in Figs. 8 and 9 with ξ and Pr as parameters, respectively. The increase of $Nu/Re_{\gamma}^{1/2}$ with relative velocity parameter γ, shown in these figures, implies that for a specified free stream the faster the moving plate the larger the heat transfer rate. Figure 8(b) shows that $Nu/Re_s^{1/2}$ increases with decreasing γ. This means that for a fixed plate velocity the heat transfer rate increases as the free stream velocity increases. Concerning the effect of Prandtl number, Fig. 9 shows that $Nu/Re_{\infty}^{1/2}$ increases as Pr increases.

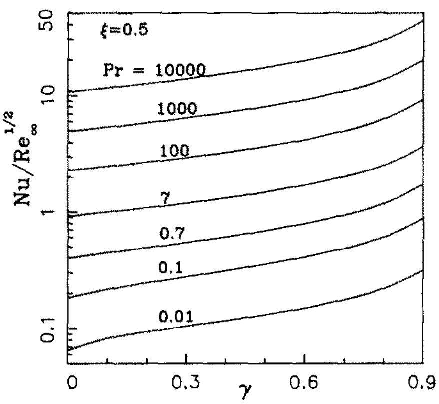

FIG. 9. Effects of Prandtl number on $Nu/Re_{\infty}^{1/2}$ for buoyancy assisting flow on a parallel moving plate, $\xi = 0.5$.

The variations of $Nu/Re_{\infty}^{1/2}$ with the mixed convection parameter ζ for some specified relative velocity parameter γ are shown in Fig. 10 for the cases of parallel and reverse moving plates. The two cases have the similar tendency of heat transfer variation with respect to ζ, as can be seen from this figure. It is seen that $Nu/Re_{\infty}^{1/2}$ increases as ζ increases for the assisting flow, but decreases with increasing ζ for the opposing flow. This figure also shows that there are three different regions: (1) a forced convection region where $Nu/Re_{\infty}^{1/2}$ is independent of ζ for small values of ζ; (b) a free convection region where $Nu/Re_{\infty}^{1/2}$ are proportional to ζ for large values of ζ; and (3) a true mixed convection region between the above two.

5. CORRELATIONS OF HEAT TRANSFER RATES

A very comprehensive correlation equation for convenient estimation of mixed convection heat transfer rate between a horizontally moving plate and a parallel free stream is proposed, based on the forced and the natural convection solutions, as

$$\left(\frac{Nu}{\lambda}\right)^m = \left((1-\xi)\frac{Nu_F}{\lambda_F}\right)^m + \left(\xi\frac{Nu_N}{\lambda_N}\right)^m. \quad (32)$$

The natural convection Nusselt number has been reported [6] as

$$\frac{Nu_N}{\lambda_N} = 0.456\left(\frac{1+Pr}{0.313+0.856Pr^{1/2}+Pr}\right)^{1/5} \quad (33)$$

with maximum error within 0.5% for $0.001 \leqslant Pr \leqslant 10\,000$ when compared with the numerical data.

The forced convection Nusselt number can be calculated from our previously developed correlation equation [13]

$$\left(\frac{Nu_F}{\lambda_F}\right)^n = \left(\gamma\frac{Nu_{F(\gamma=1)}}{(\sigma\,Re_s)^{1/2}}\right)^n + \left((1-\gamma)\frac{Nu_{F(\gamma=0)}}{(\omega\,Re_x)^{1/2}}\right)^n \quad (34)$$

which is based on the Nusselt numbers of the two limiting cases: $Nu_{F(\gamma=0)}$ for the forced convection of a stationary plate in a free stream ($\gamma = 0$); and $Nu_{F(\gamma=1)}$ for the forced convection of a moving plate in a quiescent ambient fluid ($\gamma = 1$). The best fitting of the exponent n and the maximum error of the correlation equation (34) over the entire range of relative velocity have been presented in ref. [13].

By substituting the correlation equations (33) and (34) into the comprehensive mixed convection correlation equation (32) and comparing with the numerical data, we are able to determine the exponent constant m in this equation. The appropriate values of m, n and the maximum error of the correlation equation (32) over the whole domains of mixed convection and relative velocity for $0.01 \leqslant Pr \leqslant 10\,000$ are presented in Table 1. For such a very comprehensive correlation, a maximum error of 6.5% is satisfactory.

For the special case of mixed convection between a stationary isothermal horizontal plate and a free stream ($\gamma = 0$), a reduced correlation equation from equation (32) can be written as

$$\left[\frac{Nu}{(\omega\,Re_x)^{1/2}+(\phi\,Ra)^{1/5}}\right]^m = \left[(1-\xi)\frac{Nu_{F(\gamma=0)}}{(\omega\,Re_x)^{1/2}}\right]^m + \left[\xi\frac{Nu_N}{(\phi\,Ra)^{1/5}}\right]^m. \quad (35)$$

The maximum deviation of this correlation with $m = 4$ from the numerical solution does not exceed 2.6% over the entire mixed convection regime ($0 \leqslant \xi \leqslant 1$) for a very wide range of Prandtl number ($0.001 \leqslant Pr \leqslant 10\,000$).

Table 1. Values of m, n, and the maximum error of the correlation equation (32) over the entire domains of relative velocity and mixed convection

Range of Pr	m	n	Maximum error (%)
$0.1 \leqslant Pr \leqslant 0.7$	2.55	0.87	6.5
$0.7 \leqslant Pr \leqslant 10\,000$	3.25	0.97	5.2

For the mixed convection from an isothermal horizontal plate moving continuously in a quiescent ambient fluid ($\gamma = 1$), the reduced correlation equation is

$$\left[\frac{Nu}{(\sigma\,Re_s)^{1/2}+(\phi\,Ra)^{1/5}}\right]^m = \left[(1-\xi)\frac{Nu_{F(\gamma=1)}}{(\sigma\,Re_s)^{1/2}}\right]^m + \left[\xi\frac{Nu_N}{(\phi\,Ra)^{1/5}}\right]^m. \quad (36)$$

When compared with the numerical results, the maximum error of this correlation with $m = 3.5$ is less than 2.2% over the entire mixed convection range ($0 \leqslant \xi \leqslant 1$) for $0.01 \leqslant Pr \leqslant 10\,000$.

6. CONCLUSIONS

In this paper, we have studied theoretically a general mixed convection problem of an isothermal horizontal plate moving in parallel or reversely to a free stream. By introducing proper parameters of mixed convection and relative velocity, we are able to obtain a set of universal formulation which is readily reducible to the equations of all the possible laminar convection systems on a horizontal flat plate. Very accurate numerical solutions and a comprehensive correlation equation have been presented over the whole domains of mixed convection intensity and relative velocity for a very wide range of Prandtl number between 0.01 and 10 000. Heat transfer predictions from the correlation match the numerical data to within 6.5% over the entire domains of relative velocity and buoyancy.

The effects of the buoyancy and the relative velocity between the plate and the free stream on the flow field, the surface friction, and the heat transfer rate are clearly shown. Typical temperature profiles are also presented. It is found that the heat transfer rate increases significantly with increasing the buoyancy and the velocities of the moving plate and the free stream.

The numerical results and correlations are very useful for the design and operations of several manufacturing processes such as hot rolling, extrusion, and material cooling on a conveyer.

Acknowledgement—The support of this work by the National Science Council of Republic of China is gratefully acknowledged (Grant No. NSC81-0402-E008-01).

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